Rheology-based sea ice dynamics: from the fluid-like to the state-of-the-art solid-like brittle approach

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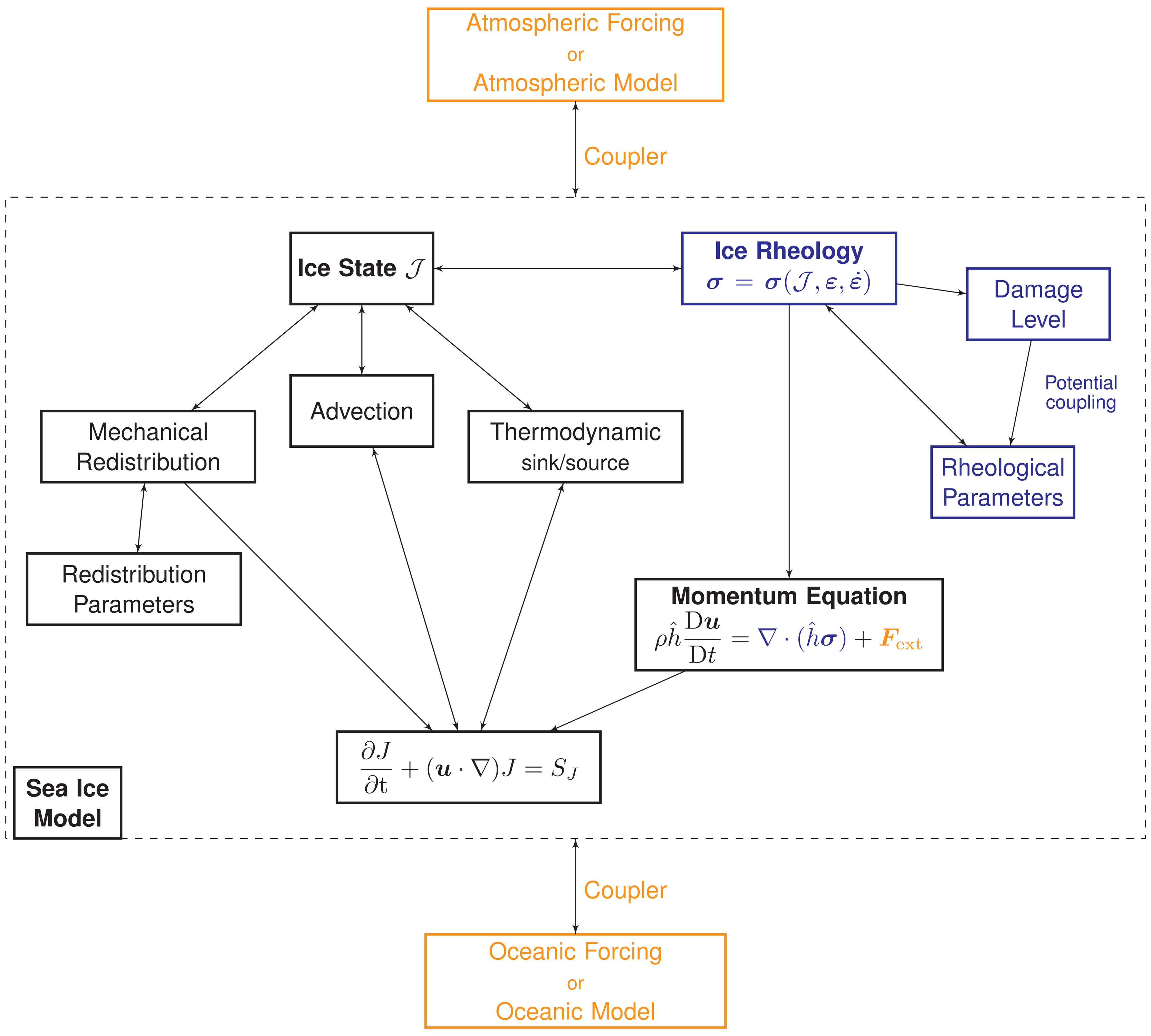
Introduction

The increasing interest in the climate in general and, in particular, in the role that Arctic processes play within it has led to an increasing demand for accurate predictions for sea ice **motion** and **deformation**, i.e. its dynamics. Finding suitable equations to describe the sea ice dynamics has been a long–standing challenge for the sea ice community. The determination of an appropriate constitutive relation, in particular, has been a demanding task. This work reviews the developments in continuum-based sea ice modeling with a particular focus on the formulation of internal stresses, i.e. the **rheology**, going from early fluid-like models to the most recent $\mathrm{neXtSIM}_{DG}$ model employing the **solid**–**brittle** rheology.

A model of sea ice dynamics

- Ice state variable $J \in \mathcal{J} = \{A, \hat{h} \dots\}$, where $\mathcal{J}$ is **model-dependent**
- Sea ice mean thickness $\hat{h}$ and concentration A being the most important
- Sea ice internal stress $\boldsymbol{\sigma}$, strain ε, strain rate $\dot{\varepsilon}$
- Equation of motion is depth-integrated



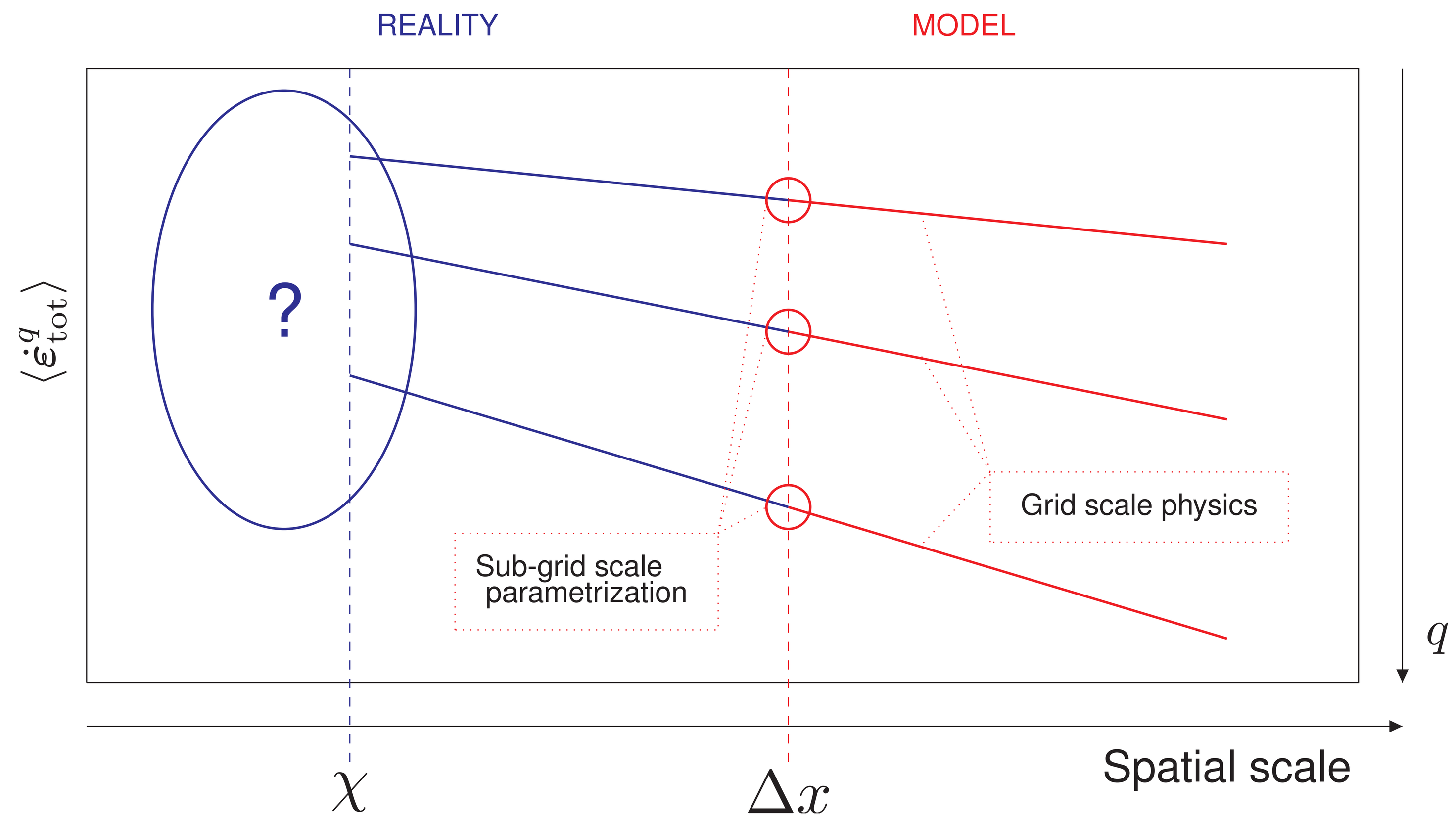
- $\boldsymbol{F}_{\mathrm{ext}} = \underbrace{A(\boldsymbol{\tau}_a + \boldsymbol{\tau}_w)}_{\text{air-water stress}} - \underbrace{\rho\hat{h}f_c\hat{\boldsymbol{k}} \times \boldsymbol{u}}_{\text{Coriolis}} - \underbrace{\rho h g \nabla H}_{\text{gravity}}$
- State variables may follow a conservation/transport law

Fluid-like models

- First efforts to model sea ice rheological behavior: **viscous fluid** model (poor approximation);
- Compact ice deformates sporatically and irreversibly $\rightarrow$ critical stress states specified by a **yield curve**
- **Plastic** rheologies were developed and coupled with different subcritical behaviors:
  - **elastic** (EP model, Coon et al. 1974) $\quad \boldsymbol{\sigma} = E\mathbf{K} : \boldsymbol{\varepsilon}$
  - **viscous** (VP model, Hibler 1979) $\quad \boldsymbol{\sigma} = 2\eta\dot{\boldsymbol{\varepsilon}} + [\zeta - \eta]\mathrm{tr}\,\dot{\boldsymbol{\varepsilon}}\mathbf{I} - \frac{P}{2}\mathbf{I}$
- A flow rule specifies how deformation occurs once a critical stress is reached

Multifractality of sea ice deformation

- Observed spatial/temporal scaling of sea ice deformation
- **Intermittency** and **heterogeneity** of deformation



Solid-like models

- Sea ice cover modeled as a progressively damaging medium (with a Mohr-Coulomb criterion). Damage level $d \leq 1$; $E = E(d), \eta = \eta(d), \lambda = \eta/E$
- **Elasto-brittle (EB)** $\quad \frac{\mathrm{D}\boldsymbol{\sigma}}{\mathrm{D}t} = E(\mathbf{K} : \dot{\boldsymbol{\varepsilon}}) - \frac{\dot{d}}{1-d}\boldsymbol{\sigma}$
- **Maxwell-Elasto-brittle (MEB)** $\quad \frac{\mathrm{D}\boldsymbol{\sigma}}{\mathrm{D}t} = E(\mathbf{K} : \dot{\boldsymbol{\varepsilon}}) - \frac{\boldsymbol{\sigma}}{\lambda}\left(1 + \lambda\frac{\dot{d}}{1-d}\right)$
- **Brittle-Bingham-Maxwell (BBM)** $\quad \frac{\mathrm{D}\boldsymbol{\sigma}}{\mathrm{D}t} = E(\mathbf{K} : \dot{\boldsymbol{\varepsilon}}) - \frac{\boldsymbol{\sigma}}{\lambda}\left(1 + \tilde{P} + \lambda\frac{\dot{d}}{1-d}\right)$

A timeline

2023	neXtSIMv2-NEMO First coupled ocean-sea ice model implementing a brittle rheology (BBM) to determine the mass balance in a pan-Arctic simulation.	Boutin et al. (2023)
2022	Brittle Bingham-Maxwell rheology (BBM), neXtSIM v2 Further evolution of MEB rheology. Addition of a compressive threshold for the subcritical elastic response.	Ólason et al. (2022)
2021	neXtSIMv1 LKFs statistics analysis Analysis of lead fraction statistics in the Central Arctic as predicted by neXtSIMv1.	Ólason et al. (2021)
2019	neXtSIMv1 scale-invariance analysis Evaluation of neXtSIM multi-fractal properties through temporal and spatial scale analysis.	Rampal et al. (2019)
2016	neXtSIMv1 Full neXtSIM stand-alone Lagrangian dynamical-thermodynamical model.	Rampal et al. (2016)
2016	Brittle Maxwell-EB (MEB) Evolution of EB-rheology. Addition of a viscous-like relaxation term, with a damage-evolving effective viscosity.	Dansereau (2016)
2015	neXtSIM sea-ice model(dynamical core) Presentation of the dynamical core of neXtSIM sea ice model using an optimized version of the EB rheology.	Bouillon and Rampal (2015b)
2011	Elasto-brittle rheology(EB) First rheology employing brittle mechanics through a progressive damage mechanism for a continuum elastic sea ice model.	Girard et al. (2011)
2000s	Evidence for a brittle sea-ice rheology Observation of sea-ice mechanical events at different scales in the RGPS data set. Analysis of spatial/temporal scaling and events localization. Analogy with rock mechanics.	Marsan et al. (2004) Weiss and Marsan (2004) Weiss et al. (2007) Rampal et al. (2008) ...

Summary

- In viscous-fluid models the mechanical parameters (η, E, λ) are independent of the internal stresses. This resulted in unphysical behaviour.
- In plastic models ice dynamic parameters adjust to keep stresses (sub)critical. However, these models fail to reproduce the observed scaling laws (multifractality).
- Brittle models, in which ice dynamic parameters do not instantaneously relax back after stresses are released, perform better at this point.

KEY REFERENCES

See full-text for the complete bibliography.

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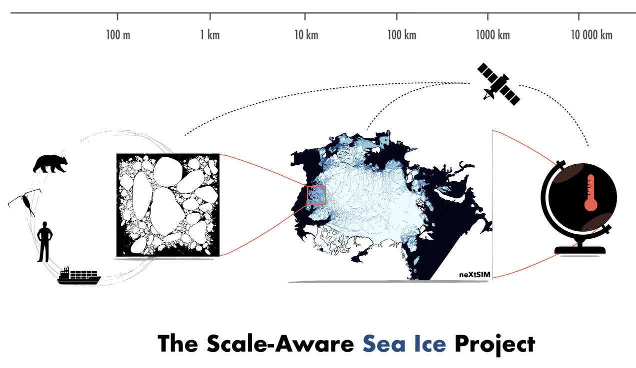

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